Effectiveness Evaluation Analysis of Weapon Equipment System Based on Weighted Gray Relational Degree

Guoqing Huang[1,2], Lin Su[2]

[1]Department of Aeronautic Electronic,The First Aeronautical College of Air Force, Xinyang, China

[2]School of Information Engineering, Zhengzhou University, Zhengzhou, China

hguoqing@163.com, 60667527@qq.com

Abstract

In order to solve the diverse problems of effectiveness evaluation of current weapon equipment systems, which are multiple hierarchies, multiple factors and different relational degree among the factors of system, the shortage of Analytic Hierarchy Process (AHP) and traditional Gray Relational Degree is analyzed and a new evaluation method of the weighted gray relational degree is proposed in this paper. This method is used for evaluating weapon equipment systems by combining AHP with Gray Relational Degree. Finally, feasibility and effectivenessof Weighted Gray Relational Degree for multiple hierarchies and multiple factors weapon equipment systems can be proved by a case analysis.The result showed that Weighted Gray Relational Degree would be more reliable than single weight of AHP. And Weighted Gray Relational Degree can solve the weakness of traditional Gray Relational Degree which isunable to effectively evaluate the multiple hierarchies and multiple factors of weapon equipment systems.

Keywords

Weapon Equipment Systems; AHP; Weighted Gray Relational Degree

Introduction

The modern effectiveness evaluation of weapon equipment systems from single function to complex structure have been paid attention all over the world with technology development. Modern equipment system model which contains a lot of fuzzy and uncertainties are mostly complex, multiple hierarchies and multiple factors. It is important to effectively evaluate modern equipment system.

There are various methods used for effectiveness evaluation at present, including AHP, Fuzzy Synthesize Analysis, Gray theory and so on. Although AHP can preferably solve the problem of multiple hierarchies system evaluation and has been used widely, subjectivity is relatively strong. In addition, influence to system efficiency by relational degree among indexes has been ignored in AHP. The method of Gray Relational Degree (GRD) can overcome this shortage. The GRD use degree between compared sequences and standard sequence to compare and sort compared sequences. It can commendably solve the uncertainissues existing in the system. But traditional GRD has been unable to effectively evaluate multiple hierarchies system model. So GRD would be combined with other methods.

The Weighted Gray Relational Degree by combining AHP with GRD, therefore, is proposed to evaluate weapon equipment systems of multiple hierarchies and multiple factors in this paper. A specific case of effectiveness evaluation of airborne command and control system was presented and it proved reasonability and effectivity of Weighted Gray Relational Degree.

Calculation of Index Weights

Establish Index System

It is the foundation to establish evaluation index system for effectiveness evaluation. Generally, index system contains goal hierarchy,criterion hierarchy and index hierarchy. In this paper, the airborne command and control system from literature [1] used. The evaluation goal is system effectiveness. There are four criteria which are B1, B2,

B3 and B4 in criterion hierarchy. Meanwhile, the index hierarchy contains sixteen indexes which are C11~C15, C21~C24, C31~C34 and C41~C43.

Index Weights Calculation by AHP

1) Establish Pairwise Compared Matrix

It is used a 1~9 scale measurement to compare elements reciprocally one by one and established pairwise compared matrix represented as A.The 1~9 scale measurement is represented in Table 1.

There are two types of pairwise compared matrix which are goal-criterion and criterion-index matrix.The form of these two types would be same despite of hierarchy difference. The specific form of compared matrix is presented in Equation(1).

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \tag{1}$$

Where A represents pairwise compared matrix and $a_{i,j}$ means the relative value of *ith* criterion and *jth* criterion.

TABLE 1:1~9 SCALE MEASUREMENT

Value	Significance
1	Index i and index j are equally important
3	Index i is sightly important than index j
5	Index i is obviously important than index j
7	Index i is quite important than index j
9	Index i is absolutely important than index j
2, 4, 6, 8 are the middle value ; If index i is less important than index j, value would be 1/v,where v is1~9	

2) Calculation of Index Weights

Calculation of index weights in AHP can be placed under calculation of maximum eigenvalue and eigenvector. There are many calculation methods at present, such as mean square method, sum and product method,power method and so on. Mean square method is used in this paper and presented as follows.

Step 1:Calculating product of each row elements showed in Equation(2).

$$M_i = \prod_{j=1}^{n} B_{ij}, i = 1, 2, \cdots, n \tag{2}$$

Step 2:Calculating n times mean square of M_i showed in Equation(3).

$$\overline{w_i} = (M_i)^{\frac{1}{n}}, i = 1, 2, \cdots, n \tag{3}$$

Step 3:Normalizing $\overline{w_i}$ showed in Equation(4)

$$w_i = \frac{\overline{w_i}}{\left(\sum_{i=1}^{n} \overline{w_i}\right)}, i = 1, 2, \cdots, n \tag{4}$$

So weight vector needed can be represented as:

$$w = [w_1, w_2, \cdots, w_n]^T .$$

Step 4:Calculating maximum eigenvalue as λ_{max} showed in Equation(5).

$$\lambda_{\max} = \sum_{i=1}^{n} \frac{[Aw]_i}{nw_i} \qquad (5)$$

Where $[Aw]_i$ represents the *ith* element in vector showed as $[Aw]$.

Step 5:Testing consistency of pairwise compared matrix showed in Equation(6) and(7).

$$CI = \frac{\lambda_{\max} - n}{n-1} \qquad (6)$$

$$CR = \frac{CI}{RI} \qquad (7)$$

Where RI means Random Average Index and CI means Consistency Index.CR represents Consistency Ratio and if CR meets CR<0.1,the compared matrix can satisfy the need of consistency.

Gray Relational Degree Analysis

GRD is the method which analyses geometry shape resemble degree of sequence curve of impact factor to estimate relational degree. That is, GRD sorts evaluation index through estimating relational degree between each compared sequences and standard sequence.

Establish Compared Sequences

Firstly,establish compared sequences written as follows:

$$X_1 = [x_1(1), x_1(2), \cdots x_n(n)] \qquad X_2 = [x_2(1), x_2(2), \cdots x_2(n)], \qquad X_m = [x_m(1), x_m(2), \cdots, x_m(n)]$$

Where $i = 1, 2 \cdots m$; $j = 1, 2 \cdots n$. $x_i(j)$ represented the value of *jth* index of *ith* compared sequence.

Normalizing Compared Sequences and Establish Standard Sequence.

It would be hard to directly use original data for evaluation because of different dimension and order of indexes during evaluation process. Therefore,original data should be normalized.Based on different attributes, index can be divided into four kinds: benefit type,cost type,certain type and internal type. All indexes belongs to benefit type in this paper. In other words, the larger index value is,the better. The method of normalization is showed in Equation(8).

$$y_i = \frac{x_i - \min_j x_i}{\max_j x_i - \min_j x_i}, i = 1, 2, \cdots m \qquad (8)$$

So normalization decision-making matrix can be represented as follows:

$$Y = \begin{bmatrix} y_1(1) & y_1(2) & \cdots & y_1(n) \\ y_2(1) & y_2(2) & \cdots & y_2(n) \\ \vdots & \vdots & \vdots & \vdots \\ y_m(1) & y_m(2) & \cdots & y_m(n) \end{bmatrix}$$

In this paper, maximum value of compared sequences would be used as standard sequence written as Y_0

$$Y_0 = [y_0(1), y_0(2) \cdots y_0(n)]$$

Calculate Gray Relational Coefficient

$$\gamma_i(j) = \frac{\min_i \min_j |x_0(j) - x_i(j)| + \rho \max_i \max_j |x_0(j) - x_i(j)|}{|x_0(j) - x_i(j)| + \rho \max_i \max_j |x_0(j) - x_i(j)|} \qquad (9)$$

Where $\gamma_i(j)$ is the gray relational coefficient,and ρ represents the distinguishing coefficient, taking the value of

0.5 in this paper. The smaller ρ, the greater distinguishing power is.So Gray relational coefficient matrix can be achieved and showed as follows:

$$E = \begin{bmatrix} \gamma_1(1) & \gamma_1(2) & \cdots & \gamma_1(n) \\ \gamma_2(1) & \gamma_2(2) & \cdots & \gamma_2(n) \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_m(1) & \gamma_m(2) & \cdots & \gamma_m(n) \end{bmatrix}$$

Where E represents Gray relational coefficient matrix

Synthetical Analysis of Ahp aAnd Grd

For traditional GRD,gray relational degree can be achieved by weighting directly Gray relational coefficiency. But every index and criterion influence on efficiency is different,this paper would combine gray relational coefficiency withweights achieved by AHP. Thus the weighted gray relational degree can be achieved and showed in Equation(10).

$$\gamma_i = \sum_{j=1}^{m} w_i \gamma_i(j) \ , j = 1, 2, \cdots, m \tag{10}$$

Where γ_i represents weighted gray relational degree of *ith* compared sequence. If index system has three hierarchies, the gray relational degree should be weighted twice.Then final weighted gray relational degree is used as evaluation of effectiveness.The larger weighted gray relational degree isthe better one. It means that the compared sequence is closed to optimal effectiveness.Otherwise, the effectiveness would be worse.On the one hand,this method would consider the influencing proportion of each index for effectiveness.On the other hand,the problem of index uncertainty can be solved and mutual relation would be considered.The evaluating result would be more effective and reasonable.A specific case is presented as follows.

TABLE 2: ESTIMATING RANKS AND SCORES

Rank	Score
Excellent	0.9~1
Very good	0.8~0.9
Good	0.6~0.8
Medium	0.4~0.6
Bad	0~0.4

Case Analysis

Effectiveness of one airborne command and control system[1] has been evaluated and analyzed in this paper. Firstly, estimating ranks and corresponding estimating scores have been established. Estimating score represents relational degree between effectiveness of airborne command and control system and optimal effectiveness. The larger estimating score is, the more adjacent to optimal effectiveness. Thus, effectiveness of airborne command and control system would be better. The specific estimating ranks and scores are showed as in Table 2.

Based on relevant data from literature [1], index weights through AHP can be calculated and showed as follows*:*

$$W = (0.36, 0.10, 0.20, 0.34), W_1 = (0.30, 0.19, 0.10, 0.26, 0.16),$$
$$W_2 = (0.22, 0.20, 0.48, 0.10), W_3 = (0.36, 0.26, 0.30, 0.08), W_4 = (0.65, 0.10, 0.25)$$

In this paper, all indexes are qualitative. So the method for quantification would use grade evaluated by experts who can reduce individual subjectivity and uncertainty. There are five experts for this case. Arithmetic average of every index's grade would be used as value. The final grades are showed in Table.3.

The compared sequences can be achieved by normalizing and showed as: (1,0.4286,0,0.7857,0.8571), (1,0,0.4667,0.8000),(1,0,0.2222,0.4444),(0.4286,1,0).Standard sequences are showed as:(1,1,1,1,1),(1,1, 1,1),(1,1,1,1),(1,1,1).

Gray relational coefficient can be calculated by using Equation (9) and showed in Table 4.

Then gray relational coefficient would be combined with weights by Equation (10). Corresponding gray relational degree of criterion level can be achieved and showed in Table 5.

In criterion level,Equation (10) would be continued in order to achieve the final weighted gray relational degree of evaluation system.The final degree after calculating is 0.6471.Estimating rank of airborne command and control system would be good according to Table 2. Therefore ,relational degree of this system and optimal effectiveness is good. The result in this paper and literature[1] are the same. It proves that weighted gray relational degree can be effective and feasible. And other weapon equipment system can also use this method for effectiveness evaluation.

TABLE. 3 THI FINAL GRADES OF INDEXES

C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}
70	54	42	64	66	72	42	56
C_{24}	C_{31}	C_{32}	C_{33}	C_{34}	C_{41}	C_{42}	C_{43}
66	88	70	74	78	80	88	74

TABLE 4: GRAY RELATIONAL COEFFICIENT OF INDEXES

C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{21}	C_{22}	C_{23}
1	0.4667	0.3333	0.7	0.7771	1	0.3333	0.4839
C_{24}	C_{31}	C_{32}	C_{33}	C_{34}	C_{41}	C_{42}	C_{43}
0.7143	1	0.3333	0.3913	0.4737	0.4667	1	0.3333

TABLE 5:GRAY RELATIONAL DEGREE OF CRITERION LEVEL

B1	B2	B3	B4
0.7283	0.5904	0.8019	0.4867

Conclusion

Weighted Gray Relational Degree , which is achieved by AHP and GRD, is proposed for effectiveness evaluation of weapon equipment systems which are multiple hierarchies and multiple factors in this paper.And in case analysis, effectiveness and feasiblity of Weighted Gray Relational Degree is proved by evaluating Airborne command and control system.Analysis result showed that Weighted Gray Relational Degree can overcome the shortage of traditional GRD which is unable to evaluate the structure of multiple hierarchies systems.And it also can make up for fault of AHP which has failed to consider mutual relational degree among indexes.This method can also used for other weapon equipment systems of multiple hierarchies and multiple factors.

REFERENCES

[1] Wang Cheng,Wang Rui,Qi Bo-hui,"Application of Fuzzy AHP in Effectiveness Evaluation for Airborne Command and Control System",Command Control & Simulation, Vol.6,Dec.2008,pp.63-66.

[2] He Hong-guang,Wang Ping,Zhou Xiao-qing,Liu Hai-xu,Cheng Dong-xia,Cai Hong-fei,"Application of Grey Multi-hierarchy Comprehensive Evaluation in Technical Performance Evaluation of Photovoltaic power Plants ", North China Electric Power,,vol.03,2014,pp.28-34

[3] Liu Xiao-ling, Xie Xian-bin,Zhang Jia-lu,Liu Ling-li," Network Security Assessment Based on Comprehensive Grey Relational Analysis ", 2014,pp.44-49

[4] Liu Bo,Xiao Chang-lai,Tian Hao-ran,Qiu Shu-wei," Application of method Combining Grey Relational with Analytic Hierarchy Process for Groundwater Quality Evaluation in Jilin City", 2014,pp.26-29

[5] Wang Qun-feng,Jin Jia-jia,Mi Chuan-min,Fang Zhi-geng," Maximum entropy configuration model of objective index weight based on grey correlation deep coefficient ", Control and Decision, Feb. 2013, pp.235-240

[6] Wu Jing,"Research on Trajectory Planning and Effectiveness Evaluation for Multi-UAV Cooperative", Jun. 2012

Huang Guoqing, 1963, Associate Professor, Xinyang China

Su Lin, 1991, student, Zhengzhou China